NOTATION

V, velocity; P, pressure; ρ, density; x, r, θ, axial, radial, and angular cylindrical coordinates; τ_r, shear stress with respect to a surface perpendicular to r in the direction of the velocity vector V; τ_{rx}, $\tau_{r\theta}$, projections of τ_r onto the x axis and the perpendicular to the x, r plane; d_h, hydraulic diameter of the channel; G, mass flow rate; F_x, cross-sectional area of the channel perpendicular to the x axis; ξ, hydraulic resistance coefficient; φ, angle between the vector of the average gas velocity and the x axis; $\zeta_x = f(Re_x)$, the quantity ξ for axial flow.

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LAMINAR FLOWS OF A CONDUCTIVE LIQUID BETWEEN POROUS DISKS IN A TRANSVERSE MAGNETIC FIELD

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UDC 532.517.2:538.4

We investigate the influence of a transverse magnetic field on the self-similar axisymmetric flows of a viscous conductive liquid between permeable disks.

The flow of a conductive liquid in a plane channel with permeable walls under the influence of a transverse magnetic field has been studied in [1]. A self-similar solution of the problem valid for any Hartmann numbers M was obtained in the form of a regular expansion in powers of a small parameter; the parameter used was the Reynolds number R, calculated on the basis of the velocity of injection or suction. Later studies [2, 3] dealt with the corresponding flows in the case of intensive symmetric bilateral injection, with large negative values of R and small values of the parameter $\mu = M^2/R$. An asymptotic analysis of the effect of the magnetic field on flows in a plane channel that were produced by introducing the conductive liquid through one of the walls and removing it through the other, in nearly asymmetric regimes, was carried out in [4].

The behavior of a conductive liquid that occupies a half-space bounded by a rotating permeable disk (of infinite radius) was studied in [5], where, in particular, it was shown that there exist self-similar flows whose radial velocity component u and axial velocity component w, as in the case of flows of a nonconductive viscous liquid around an impermeable rotating (or motionless) disk, discovered more than 60 years ago by Karman [6], can be represented in the form

$$u = r\omega F(\zeta_*),\ w = \sqrt{\nu\omega}\, H(\zeta_*),\ \zeta_* = z\sqrt{\omega/\nu}. \tag{1}$$

The flows considered here are caused not by the rotation of the disk but by the suction or injection through the permeable walls. Therefore, the quantity we have used here as the

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Translated from Inzhenerno-Fizicheskii Zhurnal, Vol. 48, No. 4, pp. 574-581, April, 1985. Original article submitted April 2, 1984.

0022-0841/85/4804-0420$09.50

scale of velocity is w, equal to the axial velocity on one of the disks, i.e., as in the analysis of flows of a nonconductive liquid [7-10]

$$u = -\frac{1}{2}\frac{r}{h}|W|f'(\zeta),\ w = |W|f(\zeta),\ \zeta = \frac{z}{h}. \tag{2}$$

Here 2h is the distance between the disks. As in [1-5], the influence of the ponderomotive forces is taken into account in the magnetohydrodynamic approximation. Moreover, it is assumed that the electromagnetic field induced by the motion of the conductive liquid is negligibly small; the magnetic Reynolds number Re_σ is also small.

Thus, in the case under consideration, it is sufficient to add to the Navier–Stokes equations a term $\bar{j} \times \bar{B} = -\sigma B^2 u(r, \zeta)\bar{r}^0$, characterizing the mass ponderomotive force, i.e., to write

$$u\frac{\partial u}{\partial r} + w\frac{\partial u}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial r} - \frac{1}{\rho}\sigma B^2 u + \nu\left\{\frac{\partial}{\partial r}\left[\frac{1}{r}\frac{\partial}{\partial r}(ru)\right] + \frac{\partial^2 u}{\partial z^2}\right\},$$

$$u\frac{\partial w}{\partial r} + w\frac{\partial w}{\partial z} = -\frac{1}{\rho}\frac{\partial P}{\partial z} + \nu\left[\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w}{\partial r}\right) + \frac{\partial^2 w}{\partial z^2}\right], \tag{3}$$

$$\frac{\partial u}{\partial r} + \frac{u}{r} + \frac{\partial w}{\partial r} = 0.$$

Consequently, the unknown function $f(\zeta)$ must satisfy the equation

$$f''' - R\left(ff'' - \frac{1}{2}f'^2\right) - M^2 f' = k. \tag{4}$$

If w is taken on the surface of the lower disk ($z = 0$, $\zeta = 0$) and is considered positive for injection, and the axial velocity on the surface of the upper disk ($z = 2h$, $\zeta = 2$) is $w(2h) = \chi|W|$, then the function $f(\zeta)$ must satisfy the following boundary conditions:

$$f(0) = \operatorname{sign} W,\ f'(0) = 0,\ f(2) = \chi,\ f'(2) = 0. \tag{5}$$

The second and fourth conditions reflect the absence of slippage on the surface of the disks.

The flow between the permeable disks at low suction and injection rates (small in the modulus of the Reynolds numbers R) will be the same as in flows of a conductive liquid in a plane channel with impermeable walls, which were studied by Hartmann [11] or, more precisely, of the Hele–Shaw type of flows of a conductive liquid in slit-type channels [12]. In connection with this, it is interesting to consider flows corresponding to high suction and injection rates ($|R| \gg 1$).

As was shown in [9], the nature of the asymptotic behavior (the number, thickness, and position of boundary layers) of self-similar flows of a nonconductive liquid between permeable disks depends on the numbers R and $R_1 = \chi|R|$. To illustrate the influence of the magnetic field, we should select regimes included in region 2 of the diagram constructed in [9]. The flows corresponding to this region are flows which arise in the case of intensive suction through one disk (for example the lower disk) and injection at any intensity through the other (upper) disk, so that $R \gg 1$, $R_1 \ll 0$. These regimes are distinguished by the fact that the distribution of velocities along the height of the gap is most nonuniform when $M = 0$ and in the limit ($R \to \infty$) we obtain a linear distribution corresponding to the line segment connecting the points $\zeta = 0$, $f' = 1$ and $\zeta = 2$, $f' = 0$. It is natural to expect that such flows will be particularly sensitive to the apparent "magnetic viscosity" caused by the action of the transverse magnetic field on the conductive liquid.

The application of a magnetic field leads to the separation of regimes which are qualitatively uniform for $M = 0$ into two classes which, by analogy with the flows of a nonconductive liquid in a plane channel, considered in [13], can be designated as "mixed suction" and "mixed injection," respectively.

When $k = \text{const}$, Eq. (4) can be simplified by differentiating with respect to ζ. As a result, for large values of R we obtain the following singularly perturbed equation:

$$ff''' + \mu f'' - \varepsilon f^{IV} = 0. \tag{6}$$

Here

$$\mu = M^2/R, \ \varepsilon = 1/R. \tag{7}$$

When the inequalities

$$\varepsilon \ll \mu \ll 1 \tag{8}$$

are satisfied, the solution of the shortened equation

$$ff''' + \mu f'' = 0, \tag{9}$$

according to the terminology used in [14], will be stable to the right. Therefore, as in the case M = 0 (μ = 0), considered in [9], a boundary layer having a thickness of the order R^{-1} and characterized by a steep increase of the radial component of the velocity, will exist only at the lower disk, and the problem can be solved by splicing the outer and inner asymptotic expansions. The external solution for the region $0 < \zeta \leq 2$ is sought in the form of an expansion in integral powers of the small parameters ε and μ:

$$f^{(0)} = f_0 + \mu^2 f_{12} + \ldots + \varepsilon f_1 + \varepsilon^2 f_2 + \ldots \quad . \tag{10}$$

It must satisfy the last two boundary conditions of (5), i.e.,

$$f^{(0)}(2) = \chi, \ f^{(0)\prime}(2) = 0. \tag{11}$$

A suitable asymptotic expansion of the solution in the interval $|0, 2|$ can be written in this case in the form

$$f(\zeta, \mu, \varepsilon) = f^{(0)}(\zeta, \mu, \varepsilon) + \varepsilon \Pi_1 f(\eta) + \ldots \tag{12}$$

Here we use the elongation transformation $\eta = \zeta/\varepsilon = R\zeta$ at the point $\zeta = 0$; the boundary functions are $\Pi_j f(\eta) \to 0$ as $\eta \to \infty$.

By virtue of inequalities (8), the first term of expansion (10) must satisfy the equation

$$f_0''' = 0 \tag{13}$$

and the boundary conditions

$$f_0(0) = -1, \ f_0(2) = \chi, \ f_0'(2) = 0, \tag{14}$$

i.e., as in the case M = 0 [9],

$$f_0(\zeta) = \chi - \varkappa_0 \left(2 - 2\zeta + \frac{\zeta^2}{2}\right) \tag{15}$$

and, correspondingly,

$$\Pi_1 f(\eta) = 2\varkappa_0 \exp(-\eta), \ \varkappa_0 = \frac{\chi + 1}{2}. \tag{16}$$

For the second term of the external expansion, we can write the following equation:

$$\frac{d^3 f_{11}}{d\zeta^3} = -\frac{1}{f_0}\frac{d^2 f_0}{d\zeta^2} = \frac{2\varkappa_0}{-2 + 4\varkappa_0\zeta - \varkappa_0\zeta^2}. \tag{17}$$

Depending on the value of the parameter κ_0, the first integral of this equation can be represented by one of the following three formulas [15]:

$$f_{11}'' = \frac{\varkappa_0}{\sqrt{2\varkappa\chi}} \ln \frac{\sqrt{2\varkappa_0\chi} - \varkappa_0(2-\zeta)}{\sqrt{2\varkappa_0\chi} + \varkappa_0(2-\zeta)} + C_1'' \quad \begin{pmatrix} \varkappa_0 < 0 \\ \chi < 0 \end{pmatrix}, \tag{18}$$

$$f_{11}'' = \frac{2\varkappa_0}{\sqrt{-2\varkappa_0\chi}} \operatorname{arctg} \frac{\varkappa_0(2-\zeta)}{\sqrt{-2\varkappa_0\chi}} + C_1'' \quad \begin{pmatrix} 0 < \varkappa_0 < 1/2 \\ -1 < \chi < 0 \end{pmatrix}, \tag{19}$$

$$f_{11}'' = \frac{-2}{2-\zeta} + C_1'' \quad \begin{pmatrix} \varkappa = 1/2 \\ \chi = 0 \end{pmatrix}. \tag{20}$$

The first formula corresponds to flows with "mixed injection," when $\chi < -1$ ($\kappa < 0$), the second to flows with "mixed suction," when $-1 < \chi < 0$ ($0 < \kappa < 1/2$). The last formula corresponds to degenerate regimes of flow which arise when there is no injection ($\chi = 0$, $\kappa = 1/2$). The second degenerate regime $\kappa = 0$, $\chi = -1$, corresponding to the trivial solution $f_{1n}(\zeta) = 0$, is of no interest, since the magnetic field under consideration has no effect on the purely axial flow of a conductive liquid.

Thus, three qualitatively different regimes of flow must be considered separately. Double integration of the expressions (18) and (19), using the homogeneous boundary conditions

$$f_{11}(0) = 0,\ f_{11}(2) = 0,\ f'_{11}(2) = 0 \tag{21}$$

leads to the following results: 1) mixed injection

$$f'_{11}(\zeta) = \left[1 - \ln(1-4\lambda^2) + \frac{1+4\lambda^2}{4\lambda}\ln\frac{1-2\lambda}{1+2\lambda} - \lambda\ln\frac{1-\lambda(2-\zeta)}{1+\lambda(2-\zeta)}\right](2-\zeta) + \ln[1-\lambda^2(2-\zeta)^2],$$

$$f_{11}(\zeta) = \{1 - \ln[1-\lambda^2(2-\zeta)^2]\}(2-\zeta) - \tag{22}$$

$$-\left[1 - \ln(1-4\lambda^2) + \frac{1+4\lambda^2}{4\lambda}\ln\frac{1-2\lambda}{1+2\lambda}\right]\frac{(2-\zeta)^2}{2} + \frac{1+\lambda^2(2-\zeta)^2}{2\lambda}\ln\frac{1-\lambda(2-\zeta)}{1+\lambda(2-\zeta)},$$

2) mixed suction

$$f'_{11}(\zeta) = \left\{1 - \ln(1+4\lambda^2) + \frac{1-4\lambda^2}{2\lambda}\operatorname{arctg}2\lambda - 2\lambda\operatorname{arctg}[\lambda(2-\zeta)]\right\}(2-\zeta) + \ln[1+\lambda^2(2-\zeta)^2],$$

$$f_{11}(\zeta) = \{1 - \ln[1+\lambda^2(2-\zeta)^2]\}(2-\zeta) - \tag{23}$$

$$-\left[1 - \ln(1+4\lambda^2) - \frac{1-4\lambda^2}{2\lambda}\operatorname{arctg}2\lambda\right]\frac{(2-\zeta)^2}{2} - \frac{1-\lambda^2(2-\zeta)^2}{\lambda}\operatorname{arctg}[\lambda(2-\zeta)]$$

[in (22) and (23) we use the dimensionless parameter $\lambda = \sqrt{\kappa_0/2\chi}$]; 3) unilateral suction.

Formal integration of Eq. (20) yields the following expressions:

$$f'_{11}(\zeta) = 2\ln(2-\zeta) - C''_1(2-\zeta),$$
$$f_{11}(\zeta) = 2(2-\zeta)[1-\ln(2-\zeta) + C''_1\frac{(2-\zeta)^2}{2} - 4(1-\ln 2) - 2C''_1. \tag{24}$$

The last solution satisfies only the first two boundary conditions of (21). At the point $\zeta = 2$, the derivative $f'_{11}(\zeta)$ has a logarithmic singularity, and the corresponding solution can be harmonized with the last boundary condition of (21) only by introducing the special boundary function Qf, which is essential only in a small neighborhood of the point $\zeta = 2$.

Thus, as $\chi \to 0$, a moderate magnetic field ($M^2/R \ll 1$) leads to the appearance of a magnetohydrodynamic boundary layer in a neighborhood of the impermeable wall. When there is unilateral suction of a nonconductive liquid, no boundary layer is formed at this wall [9].

Figure 1a shows the profiles of the function $f'_{11}(\zeta)/\kappa$, calculated by formulas (22) and (23) for flow regimes with "mixed suction" ($\chi = -1/2$, $\kappa = 1/4$) and with "mixed injection" ($\chi = -3/2, -3$; $\kappa = -1/4, -1$). This function characterizes the perturbation of the triangular distribution of the radial velocity component (as $R \to \infty$, $f(\zeta)/\chi \to 2 - \zeta$) of a conductive liquid as a result of the application of a transverse magnetic field.

As in the flow of a conductive liquid in a plane channel with nonpermeable walls, studied by Hartmann [11], or in Hele-Shaw flows of a conductive liquid between parallel plates [12], the magnetic field leads to an apparent "magnetic viscosity," which makes possible the equalization of the radial-velocity profile along the height of the gap. Other conditions being equal, this effect is particularly sensitive as $\chi \to 0$; with "mixed suction" it is more pronounced than with "mixed injection." The particularly strong influence of the magnetic field on flows of a conductive liquid which corresponds to practically unilateral suction is due to the fact that in these regimes the main mass of the liquid enters any annular cross section

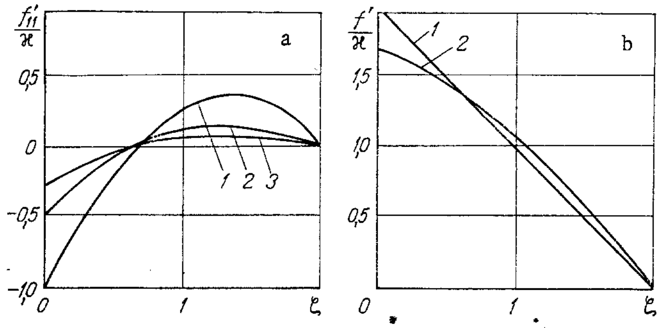

Fig. 1. a) Profiles of the functions $f'_{11}(\zeta)/\kappa$ (1, $\chi = -1/2$, $\kappa = 1/4$; 2, $\chi = -3/2$, $\kappa = -1/4$; 3, $\chi = -3$, $\kappa = -1$) and b) graph of the deformation of the triangular distribution of radial velocities of a conductive liquid along the height of the gap (1, $M^2/R = 0$; 2, $M^2/R = 1$ for $\chi = -3$, $\kappa = -1$).

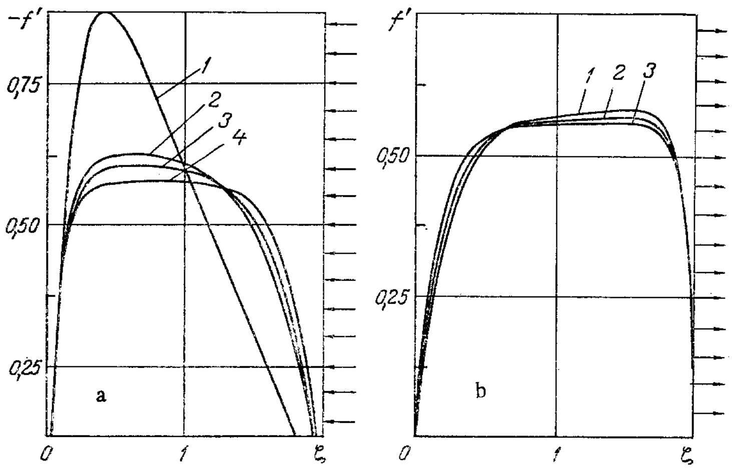

Fig. 2. Character of the distribution of the radial velocities of a conductive liquid in a magnetic field: a) under injection (1, $\mu = 0$, $R = -24$; 2, $\mu = -2.67$, $R = -26$; 3, $\mu = 3.63$, $R = -23.8$; 4, $\mu = -5.44$, $R = -25$); b) under suction (1, $\mu = 2.7$, $R = 12$; 2, $\mu = 3.2$, $R = 13.6$; 3, $\mu = 3.9$, $R = 12$).

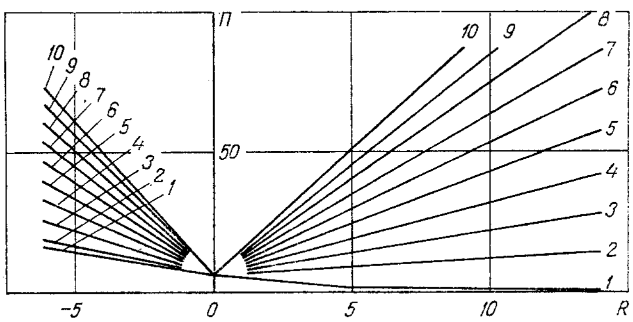

Fig. 3. Pressure gradient Π as a function of R for different magnetic-field densities: 1) $M^2/R = \pm 0$; 2) ± 0.5; 3) ± 1.0; 4) ± 1.5; 5) ± 2; 6) ± 2.5; 7) ± 3; 8) ± 3.5; 9) ± 4; 10) ± 4.5.

from the periphery, after having traveled a relatively long distance through the lines of force of the field. Figure 1b shows how the magnetic field deforms the triangular distribution of radial velocities of the conductive liquid corresponding to the limiting regimes ($R \to \infty$) along the height of the gap for relatively small values of $\mu = M^2/R$ ($\chi = -3$, $\kappa = -1$, $\mu = 1$).

In conclusion, we shall discuss the numerical analysis of those degenerate regimes of flow of a conductive liquid in an interdisk gap that are of the greatest interest. These are the flows arising in the case of unilateral injection or suction. As shown by the above case of unilateral suction ($R \gg 1$, $\chi = 0$, $\mu = 1/2$), when there are ponderomotive forces, these regimes are even more suitable for asymptotic analysis than in the case of a nonconductive liquid. On the other hand, a study of all possible self-similar flows of the type (1) with unilateral suction and injection reduces, in general, to a two-parameter analysis (this refers to the pressure gradient K and the magnetic parameter μ), for the numerical realization of which it is advisable to replace the nonlinear boundary-value problem (4), (5) with the Cauchy problem:

$$F''' - \mu_1^2 F' + \frac{1}{2} F'^2 - FF'' = K, \tag{25}$$

$$F(0) = 0,\ F'(0) = 0,\ F''(0) = (-1)^n. \tag{26}$$

Here the primes denote differentiation with respect to the new dimensionless variable $\zeta_1 = b\zeta$, and the function F is defined by $F(\zeta_1) = A^{-1}f(\zeta)$. It is assumed that

$$\frac{b}{A} = R,\ b^2A = (-1)^n,\ \frac{k}{b^3A} = K. \tag{27}$$

We assume that the lower disk ($\zeta = 0$) is nonpermeable and the Reynolds number $R = wh/\nu$ is constructed on the basis of the axial velocity on the upper disk ($\zeta = 2$ is considered positive when there is suction). For the numerical analysis of flows caused by introducing a liquid through a permeable wall, we must take $n = 1$, and the pressure gradient K must be varied between the limits 0 and ∞. In the analysis of suction, $n = 0$ and K is varied from $-\infty$ to 0.

Equation (25) is integrated to the second zero of the function $F'(\zeta_i)$: $F'(\zeta_{ij}) = 0$, $\zeta_{11} = 0$, ζ_{12} corresponds to the permeable wall, after which the main parameters characterizing the flow regime are calculated by using the formulas

$$R = \frac{1}{2}\zeta_{12}F(\zeta_{12}),\ \Pi = \frac{k\zeta_{12}}{2F(\zeta_{12})},\ M = \frac{\zeta_{12}}{2}\mu_1,\ \mu = \frac{\zeta_{12}\mu_1^2}{F(\zeta_{12})}.$$

Here the pressure gradient is $\Pi = -8h^3(\partial P/\partial x)/\rho\nu W\kappa$. The profiles of the dimensionless radial velocity component are obtained by the affine transformation $\zeta = 2\zeta_1/\zeta_{12}$, $f'(\zeta) = \zeta_{12}F'(\zeta_1)/2F(\zeta_{12})$.

Figure 2 shows how the magnetic field deforms these profiles in the case of injection or suction through the upper disk. The apparent "magnetic viscosity" facilitates the equalization of the $f'(\zeta)$ profile and the appearance at large values of the parameter $\mu = M^2/R$ of magnetohydrodynamic boundary layers on both walls of the gap. As μ increases, this layer appears first at the permeable disk in the case of injection and at the nonpermeable disk in the case of suction, i.e., at the place where for $\mu = 0$ there was no boundary layer at all, which, in particular, confirms the deduction made above in the asymptotic analysis of the degenerate case $\chi = 0$. To the limiting regimes $\mu = \pm\infty$ when $R \neq 0$ there correspond the rectangular profiles $f'(\zeta) = 1/2$.

In order to clarify the variation of the pressure gradient Π as a function of the parameter μ and the Reynolds number R, we first constructed auxiliary diagrams: $\Pi = \Pi(R, K)$, $\mu = \mu(R, K)$ for different fixed values of K. These diagrams enable us to construct the graphs of $\Pi = \Pi(R, \mu)$ for μ = const (Fig. 3). It can be seen that the magnetic field increases the hydraulic resistance both under injection and under suction, whereas for $\mu = 0$ and $R \to \infty$ it tended to zero.

NOTATION

r, z, cylindrical coordinates; 0z, axis of rotation of the disk; ω, angular velocity of the disk; ν, coefficient of kinematic viscosity of the liquid; $Re_\sigma = \sigma Bh$, magnetic Reynolds number; σ, electrical conductivity of the liquid; B, modulus of the magnetic-induction vector; h, width of the slot; P, pressure; $R = wh/\nu$, Reynolds number; $M = Bh\sqrt{\sigma/\rho\nu}$, Hartmann number; k, proportionality constant; $\bar{z}^\circ$, unit vector with radial direction.

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